RELIABILITY ASSESSMENT OF AN AIRCRAFT ENGINE ASSEMBLY: APPROFI PROJECT

P.-A. Boucard[1], D. Néron[1], N. Relun[1], B. Echard[2], N. Gayton[2], M. Lemaire[2], M. Afzali[3], F. Lefebvre[3], A. Bignonnet[4], G. Defaux[5], J-L. Dulong[6], A. Pyre[7], A. Ghouali[7], A. Otsmane[7], H. Cherouali[7]

[1]LMT Cachan (ENS Cachan/CNRS/UPMC/PRES UniverSud Paris),
61 av. du Président Wilson, F-94230 Cachan, France
Email : boucard@lmt.ens-cachan.fr, neron@lmt.ens-cachan.fr, relun@lmt.ens-cachan.fr
[2]Clermont Université, Institut Français de Mécanique Avancée, EA 3867 Laboratoire de Mécanique et Ingénieries, BP 10448, 63000 Clermont-Ferrand, France,
Email: benjamin.echard@ifma.fr, nicolas.gayton@ifma.fr
[3]Cetim, 4AB Consulting, 5Phiméca,
[6]Université de Technologie de Compiègne, [7]Snecma

Abstract. In sectors such as aerospace industry, the probabilistic approaches have now become a priority in the components' design. For this reason, French public laboratories (IFMA-LAMI, ENS-LMT CACHAN, UTC-LABORATOIRE ROBERVAL) and companies (CETIM, SNECMA, PHIMECA, MODARTT) considered as experts in their domains of interest have gathered to create the APPRoFi project – APproche mécano-Probabiliste pour la conception Robuste en Fatigue". Funded by the ANR – Agence Nationale de la Recherche, this project is based on the application of probabilistic approaches to assess the reliability of two mechanical components subjected to fatigue failure and which are designed by SNECMA. The project aims at demonstrating the interest and the contributions of such probabilistic approaches for industry.
This paper presents an overview of the methodology implemented to assess the reliability of an unmarked aircraft engine assembly that represents typical industrial problematic. The main contribution presented herein is a specific numerical strategy that has been used to reduce the computational time from several months to a few days: more than 2500 sets of parameters have been studied taking into account variations of geometrical details, bolt pretension, rotation rate, external forces, Young modulus. This sensitivity analysis allows us to identify the influential parameters and to provide a database of the results that can be used to evaluate safety margin.

1 INTRODUCTION

The assessment of reliability is nowadays a major challenge in the design of industrial structures. To obtain an accurate idea of the safety margin and to efficiently design a structure without endangering its integrity, probabilistic approaches should be used. Moreover, before managing a probabilistic approach, a sensitivity analyse should be performed to identify the most influential factors on the design. Both these studies lead to very high computational costs, as a large number of calculations must be carried out: the greater the number of parameters considered, the greater the number of calculations.
A random fatigue load is applied to the structure. It is characterised by random percentages which define the number of occurrences of each flight mission. The equivalent fatigue theory is used to represent this fatigue load by a single scalar [1]. On the other side, probabilistic fatigue curves are modelled to define the strength of the structure. In the end, the safety margin is obtained by comparing the strength and the load. To reduce the computational time, the failure probability of the structure is assessed using a numerical method called AK-IS based on Kriging and Importance Sampling. This method is derived from AK-MCS which combines Kriging and Monte Carlo Simulation [2]. This method requires several number of calculations associated to the data sampling. Again, these calculations lead to very high computational costs.
The objective of the work presented here is to develop an efficient strategy for the parametric analysis of an engine assembly. These assemblies are used in elastic structural assemblies with local nonlinearities (such as unilateral contact with friction) under quasi-static loading. Our approach is based on a decomposition of an assembly into substructures (representing the parts) and interfaces (representing the connections). The problem within each substructure is solved by the finite element method, while an iterative scheme based on the LATIN method is used for the global resolution. The proposed strategy consists in calculating the response of the assembly for several design configurations. Each design configuration corresponds to a set of values of all the variable parameters (friction coefficients, prestresses, size of geometrical details…) which are introduced into the mechanical analysis. Here, instead of carrying out a full calculation for each design configuration, we propose to use the capabilities of the

LATIN method and reutilize the solution of one problem (for one set of parameters) in order to solve similar problems (for the other sets of parameters) [3, 4, 5].
First we present the main ideas of the computational strategy. We illustrate the performance of this strategy on an academic example. The last part demonstrates the performance of the method for parametric studies on an aircraft engine assembly. Due to confidentiality reason, numerical value, geometry and mesh, load conditions can not be presented.

2 INTRODUCTION OF THE LATIN METHOD

The LATIN method uses a specific domain decomposition, splitting the structure into a set of substructures and interfaces. The specificity of this decomposition is the use of mixed unknowns on the interfaces, as illustrated on Figure 1. Consequently, the interface's unknowns are velocities and forces, which allow taking into account some complex relations such as contact, friction, cohesive behaviour...

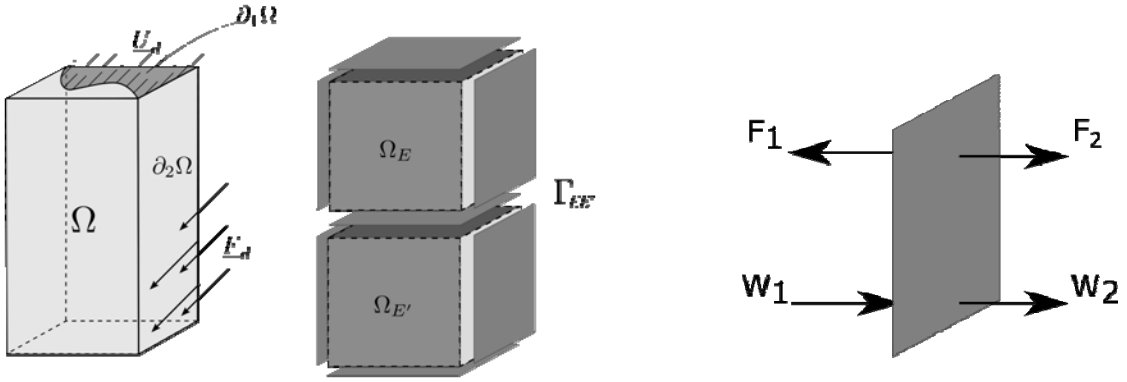


Figure 1: Domain decomposition and unknowns on interfaces

This decomposition is naturally fitted for the resolution of an assembly, the LATIN substructures become the parts and the LATIN interfaces the connections.

2.1 The problem at the substructure level

The state of the substructure Ω_E is defined by the set of fields $\{\sigma^E, \varepsilon^E,\}$ over $[0,T]$, where σ^E is the Cauchy stress tensor field, ε^E the elastic strain tensor field.

The mechanical problem to be solved in each substructure is to find the evolution over $[0,T]$ of $\{\sigma^E, \varepsilon^E\}$ subject to:

- Kinematic admissibility, which defines the space $\mathcal{U}^E$:

$$\underline{U}^E \in \mathcal{U}^E \Leftrightarrow \forall t \in [0,T], \underline{U}^E\big|_{\partial\Omega_E} = \underline{W}^E \quad (1)$$

with $\underline{U}^E$ the displacement field of the substructure and $\underline{W}^E$ the displacement on the interfaces. For the affine space $\mathcal{U}^E$, $\mathcal{U}_0^E$ denotes the associated vector space defined by:

$$\underline{U}^E \in \mathcal{U}_0^E \Leftrightarrow \forall t \in [0,T], \underline{U}^E\big|_{\partial\Omega_E} = \underline{0} \quad (2)$$

- Equilibrium, which defines the space $\mathcal{S}^E$:

$$\sigma^E \in \mathcal{S}^E \Leftrightarrow \left\{ \forall t \in [0,T], \forall \underline{U}^{E*} \in \mathcal{U}_0^E, \int_{\Omega_E} \sigma^E : \varepsilon(\underline{U}^{E*}) d\Omega_E = \int_{\Omega_E} \underline{f}_d \underline{U}^{E*} d\Omega_E + \int_{\partial\Omega_E} \underline{F}^E \underline{U}^{E*} d\Gamma \right. \quad (3)$$

(σ^E is symmetric)

- Behaviour:

$$\sigma^E = \mathbb{K}\, \varepsilon_e^E \quad (4)$$

2.2 The problem at the interface level

The state of an interface $\Gamma_{EE'}$ can be defined by the set of fields $\{\underline{F}^E, \underline{W}^E\}$. The mechanical problem is to find the evolution over $[0,T]$ such that:

- Equilibrium:

$$\forall (M,t) \in \Omega_E \times [0,T], \underline{F}^E(M,t) + \underline{F}^{E'}(M,t) = \underline{0} \quad (5)$$

- Behaviour:

$$\forall (M,t) \in \Omega_E \times [0,T], \underline{F}^E(M,t) = \mathcal{R}\left(\underline{W}^{EE'}(M,\tau), \tau \in [0,t]\right) \quad (6)$$

where $\mathcal{R}$ is a nonlinear evolution law between the forces and the velocity jump (defined by $\underline{W}^{EE'} = \underline{W}^{E'} - \underline{W}^{E}$)

Consequently, the state of the whole assembly is defined by the states of the substructures and interfaces, i.e.:

$$s = \sum_{\Omega_E \subset \Omega} s^E, \text{ with } s^E = \{\sigma^E, \varepsilon^E, \underline{F}^E, \underline{W}^E\}, t \in [0,T]$$

2.3 Separation of the difficulties

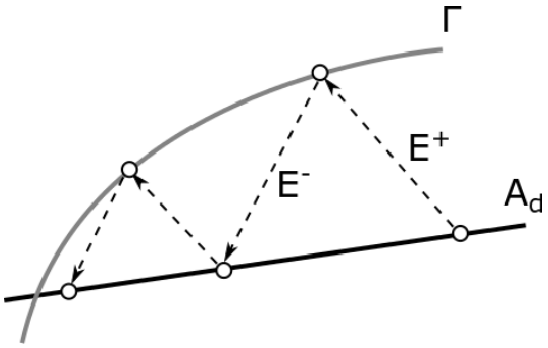
The next step of the LATIN method consists in dividing the equations into two groups in order to avoid having to solve a problem which is both nonlinear *and* global. Thus, one considers two sets of solutions:

- $\mathcal{A}_d$ is the set of the solutions s^E of the linear (possibly global) equations.
- Γ is the set of the solutions s^E of the local (possibly nonlinear) equations.

Classically, the equations on the interfaces are allocated to Γ given their local influence. The equations related to the substructures are not allocated right now, the choice will be done in the section 2.4.

2.4 The iterative scheme

Consequently, the exact global solution is the intersection of $\mathcal{A}_d$ and Γ. In order to seek this exact solution, the algorithm creates successive approximate solution s in $\mathcal{A}_d$ and Γ alternatively (see opposite Figure). It leads to the introduction of search directions E^+ and E^- and their role illustrated on the same figure, which represents the space of the solutions.



Thus, each LATIN iteration consists of two steps:

- The local step: with $s_n \in \mathcal{A}_d$, find $\hat{s}_{n+\frac{1}{2}}$ such that $\hat{s}_{n+\frac{1}{2}} \in \Gamma$ and $\hat{s}_{n+\frac{1}{2}} - s_n \in E^+$
- The linear step: with $\hat{s}_{n+\frac{1}{2}} \in \Gamma$, find s_{n+1} such that $s_{n+1} \in \mathcal{A}_d$ and $s_{n+1} - \hat{s}_{n+\frac{1}{2}} \in E^-$

The search directions are only parameter of the method and only impact the convergence rate of the method, but not its solution. Without them, the problem associated to the resolution of $\mathcal{A}_d$ leads to ill-posed problem and Γ to a problem without solution, so their introduction is necessary.

The search directions are linear equations between primal and dual unknowns, and are defined by:

$$\hat{s}_{n+\frac{1}{2}} - s_n \in E^+ \Leftrightarrow \hat{F}^E_{n+\frac{1}{2}} - \underline{F}^E_n = k_0 \left(\hat{\underline{W}}^E_{n+\frac{1}{2}} - \underline{\dot{W}}^E_n\right) \quad (7)$$

$$s_{n+1} - \hat{s}_{n+\frac{1}{2}} \in E^- \Leftrightarrow \underline{F}^E_{n+1} - \hat{F}^E_{n+\frac{1}{2}} = -k_0 \left(\underline{\dot{W}}^E_{n+1} - \hat{\underline{W}}^E_{n+\frac{1}{2}}\right) \quad (8)$$

The influence and the optimal value of k_0 on the convergence has been examined in (Ladevèze, Loiseau, & Dureisseix, A micro-macro and parallel computational strategy for highly heteregoneous structures, 2001).

Consequently, the separation of the equations becomes natural:

- $\mathcal{A}_d$ is the set of the solutions s which should satisfy Equations (1), (3), (4) and (8)
- Γ is the set of the solutions s related to interfaces (Equations (5) and (6)) and should satisfy Equation (7).

2.5 The linear stage

The linear stage consists in building $s_{n+1} \in \mathcal{A}_d$ knowing $\hat{s}_{n+\frac{1}{2}} \in \Gamma$. As a reminder, we want to solve the equations:

- Kinematic admissibility of each substructure
- Equilibrium of each substructure
- Behaviour of each substructure (elasticity)
- Search direction E^-

2.6 The local stage

The local stage consists in building $\hat{s}_{n+\frac{1}{2}} \in \Gamma$ knowing $s_n \in \mathcal{A}_d$. As a reminder, we want to solve the equations:

- Interface behaviour
- Search direction E^+

These equations respectively lead to a problem on each point of the interfaces.

2.7 Interface behaviour

The computation of the interface behaviour with the search directions [6] is addressed in [7] for contact with friction problems and for cohesive interfaces.

2.8 The error indicator

Usually an error indicator computed on each interfaces is introduced in order to estimate the residual of the interface problem after a LATIN iteration. This indicator can be written as:

$$\varepsilon^a = \frac{\sum_{\Omega_E \subset \Omega} \left(\left\| \underline{\dot{W}}_n^E - \underline{\hat{\dot{W}}}_{n+\frac{1}{2}}^E \right\|_W^{E^2} + \left\| \underline{F}_n^E - \underline{\hat{F}}_{n+\frac{1}{2}}^E \right\|_F^{E^2} \right)}{\sum_{\Omega_E \subset \Omega} \left(\left\| \underline{\dot{W}}_n^E + \underline{\hat{\dot{W}}}_{n+\frac{1}{2}}^E \right\|_W^{E^2} + \left\| \underline{F}_n^E + \underline{\hat{F}}_{n+\frac{1}{2}}^E \right\|_F^{E^2} \right)} \tag{9}$$

With the norms defined by:

$$\begin{cases} \left\| \underline{x} \right\|_W^{E^2} = \int_{\Gamma_{EE}^i} \underline{x}^T \cdot k_0 \cdot \underline{x} \cdot d\Gamma_{EE}^i \\ \left\| x \right\|_F^{E^2} = \int_{\Gamma_{EE}^i} \underline{x}^T \cdot k_0^{-1} \cdot \underline{x} \cdot d\Gamma_{EE}^i \end{cases} \tag{10}$$

This indicator can be interpreted as the distance between $\mathcal{A}_d$ and Γ along the search direction E^- for a given LATIN iteration.

2.9 Algorithm

Because the LATIN method is an iterative method, we introduce an error indicator ξ and an error criterion ϵ_ξ. This leads to the algorithm on Figure 2.

Initialisation
Loop on the substructures
! Compute and factorize all the operators
Loop on the interfaces
! Initialize with zero values
Iterate until convergence
! Global stage
! Local stage
Convergence test (energy error indicator)

Figure 2: Algorithm of the method

3 THE MULTIPARAMETRIC APPROACH

A robust design and sizing of assemblies of structures needs to take into account the variability on material parameters or friction coefficients, preloading of bolts, boundary conditions, etc. Here, the given variation range of these variables, or parameters, is decomposed into some discrete values. After that, each set of parameters defines an associated problem. Consequently, we want to solve as many problems as sets of parameters, requiring an efficient strategy to solve a lot of "similar" problems.

Our approach uses the fact that the LATIN algorithm can be initialised with any solution (usually an elastic solution) provided that it verifies the admissibility conditions. Therefore, the key is to initialise the process of one problem (associated to one set of parameters) with another one (associated to another set of parameters), assumed similar, already computed. In this manner, a first, mechanically meaningful, approximation of the solution to the problem is immediately available.
Moreover, if the parameters are only on the interfaces behaviour, the construction of the substructures operators becomes useless because we can reuse those computed previously, leading to a very quick initialisation stage of the method. If the parameter affect the geometry of the structure (diameter of a hole for example), only the operator corresponding to the substructure where the hole is located need to be rebuild.
In practical terms, in the algorithm of the Figure 2, the initialisation stage only keeps the third point (the other ones disappear) which is changed: the initialisation is not the elastic solution but the "similar problem already computed" (see Figure 3).
This approach is detailed and applied on few illustration cases in [8], [9].

Initialisation
Loop on the substructures
! Compute and factorize all the operators
Loop on the interfaces
! Initialize with zero values
Define the bound of variation of the set of parameters
Loop k = 1, 2, ... number of set of parameters
! Restore interfaces quantities previously backed up
! Iterate until convergence
! Global stage
! Local stage
! Convergence test (energy error indicator)
Save interfaces quantities for the current set of parameters

Figure 3: Multiparametric approach

3.1 A first academic example

Let us consider a simple 3D problem consisting of three stacked cubes Ω_1, Ω_2 and Ω_3. A vertical load F_1 is applied to the upper cube Ω_1 in order to create a pressure load at the contact zones. A horizontal load F_2 is applied to the middle cube in order to make it slide until it hits a rigid wall. The initial distance between the middle cube and the rigid wall is denoted j. Figure 4 shows a 2D representation of the problem.

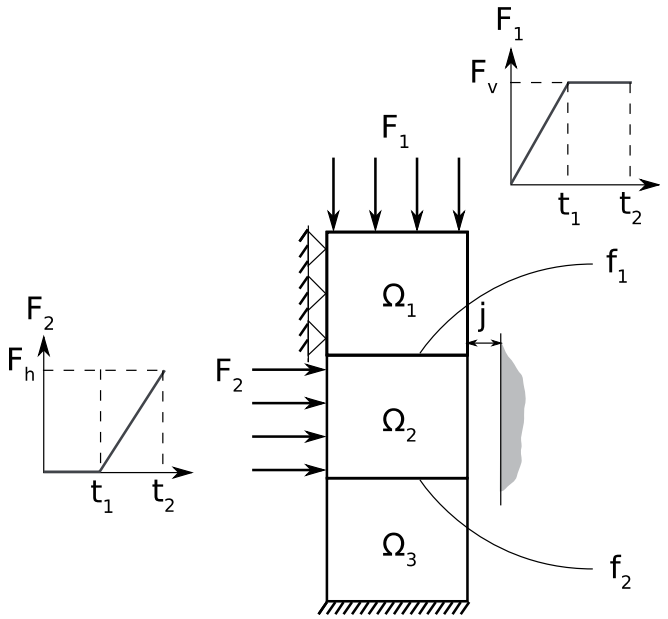


Figure 4: Model of the academic problem

As described in Figure 4, the evolution of the loads can be divided into two stages. The friction coefficients f_1 and f_2 can be different. In order to create relative movements among the cubes as soon as the compressive load is applied, different mechanical properties are assigned to each block. In particular, Poisson's coefficient is equal to 0.1 for Ω_1 and Ω_3, and equal to 0.4 for Ω_2. The objective of this problem is to evaluate the influence of the friction coefficients on the final solution. For this parametric study, f_1 and f_2 were assigned 11 different values between 0.1 and 0.6 in steps of 0.05, resulting in 121 sets of parameters for all combinations of the friction coefficients, each set defining a separate problem.

This example illustrates the purpose of the multiparametric strategy, which, as mentioned previously, is to solve many "similar" problems. The key to the method is to initialize the process associated with each similar problem (the "perturbed" structure) using the results of the calculation carried out on the "initial" structure. In this manner, a first, mechanically meaningful approximation of the solution to the perturbed problem is immediately available.

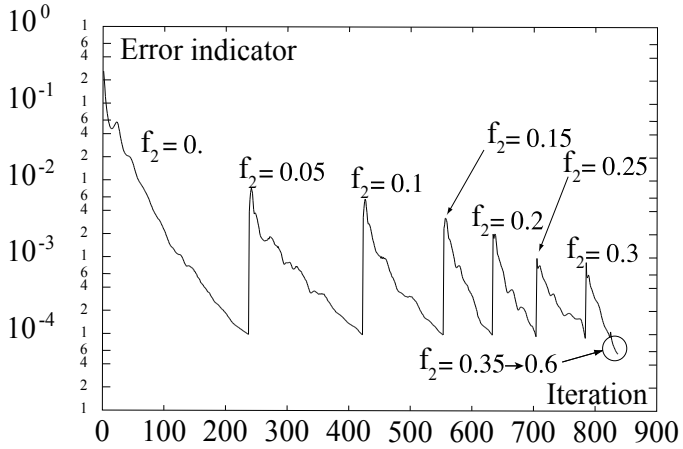


Figure 5: Evolution of the LATIN error indicator for f_1 fixed and several values of f_2

Figure 5 shows the evolution of the LATIN error indicator for several values of f2. This figure shows that the evolution of the error indicator increases every time the friction coefficient changes, but the computational cost decreased (the numbers of iterations decreased) because the results of the successive computations were very close.

Table 1 shows the computation time for one calculation, from which the computation time for 121 calculations without the multiparametric approach was extrapolated by simple multiplication. This can be used as the reference to which the running time with the multiparametric approach should be compared.

	Not multiparametric	**Multiparametric**	
	Computation time	Computation time	Gain
One set	275 s	275 s	1
121 sets	554.58 mn	6.44 mn	9.82

Table 1: Gain obtained with the multiparametric approach

Despite its simplicity, this example underlines the advantage of using the multiparametric approach with the LATIN method. Of course, the numerical implementation of this approach is easy because only the initialization of the problem needs to be changed. This leads to a substantial improvement in computation time. In the light of these results, the second example will deal with an engine assembly, which will enable us to evaluate the performance of the multiparametric approach compared to a classical finite element calculation.

3.2 Application to an unmarked aircraft engine assembly

The second example is an unmarked aircraft engine assembly that represents typical industrial problematic encountered at feasibility level. Due to confidentiality reason, numerical value, geometry and mesh, load conditions can not be presented.

This assembly is composed of 11 parts with a large number of friction and contact zones between all the parts of the assembly. Several pre-stressed bolts ensure the performance of the assembly. Total number of nodes of the model is 104 700 leading to a problem with more than 300 000 dofs. The CPU time associated to one computation is 5675 s (about one hour an half). This time will be used as reference to compare the gain obtained with the multiparametric approach. Table 2 summarizes all the computations.
One can notice that for the 2638 sets of parameters studied, the gain obtained is about 48. This result must be tempered as several parameters have a very small influence on the behaviour of the assembly. Nevertheless, this sensitivity analyse must be performed to identify the most influential factors on the design and which parameters have to be taken into account to carry out the probabilistic approach and to assess the reliability of the assembly.

	Variation considered						
	Geometrical variations		Prestress variations	Rotation rate and inertial load	External forces	Young modulus of one part	***Total***
Number of sets of parameters	441 sets (variation of 2 parameters)	52 sets (variation of 3 other parameters)	625 sets (variation of 4 parameters)	870 sets (variation of 2 parameters)	625 sets (variation of 4 parameters)	25 sets (variation of 1 parameter)	***2638 sets of parameters***
Computational time without multiparametric strategy (estimated)	about 29 days	about 82 hours	about 41 days	about 57 days	about 41 days	about 39 hours	***about 5 months and 23 days***
Computational time with multiparametric strategy	about 13 hours	about 3 hours	about 25 hours	about 25 hours	about 18 hours and 30 minutes	about 2 hours and 10 minutes	***About 3 days and 15 hours***
Gain	**53**	**27**	**39.5**	**55**	**53**	**18**	***48***

Table 2: Gain obtained with the multiparametric approach for all the parameter sets

Using this database of results, the reliability analysis was carried out only with a few random variables. The aim of this industrial application was to compute the failure probability that is the probability that the target structural performance $G(X)$ was not reached:

$$P_f = \int_{G(X)\leq 0} f_X(X)dX$$

where $f_X(X)$ is the joint density function of the random vector X . In the framework of fatigue studies, the structural performance is reached when the number of fatigue cycles exceeds a certain threshold. For practical reasons and due to random loadings as previously mentioned in introduction, the performance is formulated in equivalent stress S repeated N_{eq} times and compared to the strength R at N_{eq} cycles:

$$G(X) = R(N_{eq}, \omega) - S(N_{eq}, \omega)$$

ω refers to the randomness. The probability computation carried out by a crude Monte Carlo simulation would take about one million mechanical runs that is to say about 180 days with a classical mechanical strategy. Our proposal to couple the advanced probability computation techniques AK-IS (reducing the mechanical run number to 100 with a high confidence interval) and the proposed advanced mechanical strategy permits to evaluate the failure probability in about 7 hours. With this advanced strategy, a robust design proposal requiring several probability evaluations for different structure configurations can be planned.

4 CONCLUSION

We proposed a multiparametric computational strategy for frictional contact problems based on the LATIN method, which takes advantage of its capability to reuse the solution of a problem in order to solve similar problems. The solution of the initial problem is a very good starting point to perform calculations on other problems, provided the new conditions do not perturb the response excessively.
In this paper, we present the application of this strategy to the analysis of assemblies of elastic structures taking into account contact and friction. For these assemblies, parametric studies have been carried out on several parameters (loading, geometry, pre-stress…). We showed the drastic reduction of the computation costs.

The presented industrial example showed that the algorithm could be very efficient numerically. The parametric analysis is performed more than 40 times faster than with a direct approach.
This approach is quite general in nature and should be applicable to a number of other nonlinear problems: it has been applied to another SNECMA case (non linear viscoplastic material).

5 REFERENCES

1- Thomas J.-J., Perroud G., Bignonnet A., Monnet D., Fatigue design and reliability in the automotive industry, *European Structural Integrity Society*, 1999, 23, p. 1-11.

2- Echard, B., Gayton, N., Lemaire, M., AK-MCS: An active learning reliability method combining Kriging and Monte Carlo Simulation, *Structural Safety*, 2011, In Press: Corrected Proof.

3- Boucard, P.A., Champaney, L. A suitable computational strategy for the parametric analysis of problems with multiple contact. *Int Jal for Numerical Methods in Engineering*, 2003, 57, p. 1259-1282.

4- Boucard, P.A., Champaney, L., Guinard S. Adaptive multi-analysis strategy for contact problems with friction. Application to aerospace bolted joints. *Computational Mechanics.* Vol 42. Num 2. Pages 305-316. 2008

5- Boucard, P.A., Buytet, S., Guidault, P.A. A multiscale strategy for structural optimization. *Int Jal for Numerical Methods in Engineering.* Vol 78. Num 1. Pages 101-126. 2009

6- Blanzé, C., Champaney, L., Cognard, J.Y. et Ladevèze, P., "A modular approach to structure assembly computations", *Eng. Comput.,* Vol. 13, n°1, 15-32, 1996.

7- Kerfriden, P., Allix, O., Gosselet, P. A three-scale domain decomposition method for the 3D analysis of debonding in laminates . *Computational Mechanics.* Vol 33. Num 3. Pages 343-362. 2009

8- Roulet, V., Champaney, L., Boucard, P.A. A parallel strategy for the multiparametric analysis of structures with large contact and friction surfaces. *Advances in Engineering Software*. Vol 42. Num 6. Pages 347-358. 2011

9- Boucard, P.A., Odièvre, D., Gatuingt, F. A parallel and multiscale strategy for the parametric study of transient dynamic problems with friction. *Int Jal for Numerical Methods in Engineering*. A paraître. 2011

6 ACKNOWLEDGEMENTS

This research was supported by the Agence Nationale de la Recherche under the project named APPRoFi, which is gratefully acknowledged by the authors.